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Differential Equations: Calculus AB

Lesson Plan 2: General solution + Initial conditions ➔Particular solution.

Overview

In this lesson the students will learn how one can reduce the general solution, which contains an unknown factor (either multiplicative or additive), into a unique particular solution. This is done by the use of Initial conditions.

Learning Objectives

- Recognize multiplicity of general solution.
- Using initial condition, reducing it to particular solution.

Prior Knowledge needed

- Solving integration problems.

Instruction and activity

1. **Review + :**
    a. Homework questions from yesterday.
    b. Warm up problem: Solve

$$y = \int (x+5)dx$$

This will bring into discussion the arbitrary constant.

2. **Constant of integration:** Discussion - how the constant came about?

    This has to do with the notion of Antiderivative, which they encountered in Ch. 4.1 (page 248).

    Essentially, we were really given the following question:

    Find 'y' such that:

$$\frac{dy}{dx} = x + 5$$

And, as they talked in the past (Go specifically over how they solve it with $dy = (x+5)dx$, and then integrating both sides), y can be any of the following functions:

$$y = \frac{x^2}{2} + 5x + C$$

Because for any C, it holds true that

$$\frac{dy}{dx} = x + 5$$

C is called **Constant of Integration**.

3. **Graphically:** Let's look at this graphically, and see what the constant means:
   a. Solve $y' = x^2$.
   b. Check your answer.
   c. Plot the resulting general solution on transparency, in the form you want students to do it for the enxt question...
      ⇨ $y = \frac{x^3}{3} + C$. (C can be positive, negative, or zero)

4. **Initial conditions:** To get from this multitude of solutions to a specific one, we are given **Initial Conditions**:
   y (x=0) = 1
      ⇨ We can derive that thus C=1.
   b. And we have a **Particular solution**: $y = \frac{x^3}{3} + 1$. Note it on the graph!

5. Let's do the same for a different equation:
   a. Student solve and plot: $y' = 5y$ (HINT: use the same procedure: $\frac{dy}{y} = 5\,dx$)
      Solution: $ln(|y|) = 5x + C$ ➔ $y = e^{5x+C}$ ➔ $y = \tilde{C}e^{5x}$
   b. Compare few of the student's plots. (on transparencies)
   c. We plot together the resulting general solution. $y = \tilde{C}e^{5x}$
      ⇨ Note: $\tilde{C}$ can be positive or negative, or Zero!
   d. Given **Initial Conditions**: y (x=1) = 300
      ⇨ We can derive that thus C=2.
   e. And we have a **Particular solution**: $y = 2e^{5x}$. Note it on the graph!
   f. Note: This time it is a **multiplicative** factor.

6. Can there be other kinds of 'arbitrary' factor: We saw addition and multiplication. Others? Let students try and suggest.
   a. Present the problem: $\frac{dy}{dx} = 1 + y^2$.
   b. Let students solve, verify solution, and plot (!). { integral of 1/(1+x^2)dx = arctan(x) }
   c. Verify.

7. **Problem from AP-test:**
   (Calculus AB , Multiple choice (no calculator), 2008)

   Let students solve on their own, and the consolidate.

Which of the following is a solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

a. y=2x for x>0
b. y=2x-6 for x≠3
c. y=-sqrt(4x^2-12) for x>sqrt(3)
d. y=sqrt(4x^2-12) for x>sqrt(3)
e. y=-sqrt(4x^2-6) for x>sqrt(1.5)

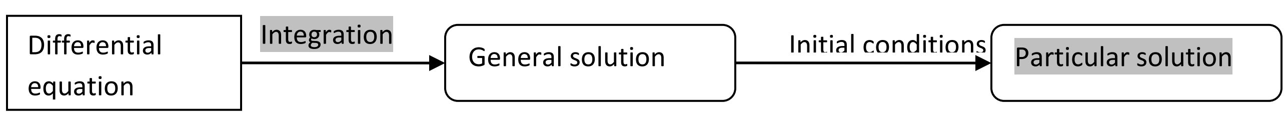
➔ **Two ways to solve: By substitution, or by solving directly.**

(Answer is c)

8. **Wrap-up :** build a graphic organizer:

Differential equation ➔ (integration) ➔ General solution (includes constant) ➔ (initial conditions) ➔ Particular solution.

(the student put the shaded ones in)



9. **Homework:** P. 409 : 31,32,36 . P. 410: 37,43,48.

====End====