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Differential Equations: Calculus AB

Lesson Plan 3: Solution curves + Slope field.

Overview

Given a general-solution to a differential equation, the students will draw the various possible solutions on a graph. Given the differential equations, the students will draw the slope field. Finally, the students will be able to connect the two, and understand the relation between those.

This is inherently a tied-together subject, but it is longer than a 1-hr lecture. Thus, it will be explained here, and most of the examples, and building insight, will be done in the next lesson.

Learning Objectives

- Given general solution, draw solution curves.
- Two ways to get to slope-fields:
  - From the derivatives of the solution curves.
  - From the differential equation.
- See the various relations between Solution-curves and Slope-field.

Prior Knowledge needed

The students must know the relation between numerical value of a slope of a line, to the angle of this line when drawn on a plot.

Special Materials

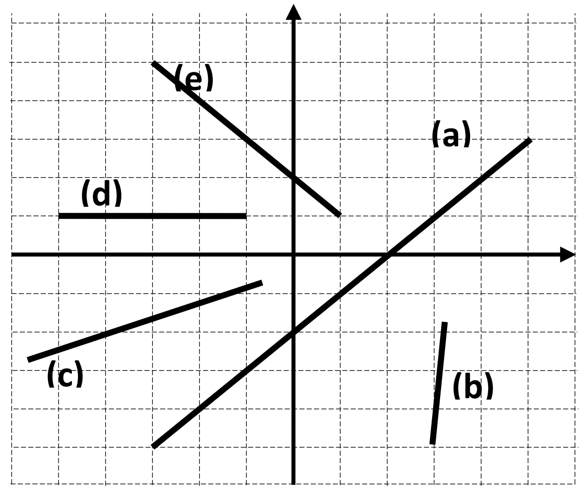
- Transparencies + Sharpies for students to draw their slope-field.

Instruction and activity

1. **MUST:** review of slope of a line and line drawing.

   Warm up question (on transparency):

   <u>Question</u>: Match between the different lines drawn and their slope.



(1) 0 ; (2) 999 ; (3) 0.4 ; (4) 1 ; (5) -1 ;

Answer: a->4 , b->2 , c->3 d->1 , e->5

➔Keep a summary of those on the Side of the board.

2. **Graphical** introduction to slope fields: (← Teacher activity)
   a. Teacher explanation of solution curves and slope fields: using two transparencies:
      i. Consider the equation $y' = 2x$. The solution is $y = x^2 + C$.
      ii. Draw on a transparency some solution curves.
      iii. Put another transparency on top:
         1. Write the equation at top left.
         2. Draw the axis on this one as well.
         3. Draw the line-segments tangential to the curve at a few points.
      iv. Then, take off the original slide, and say: This is the slope field!

3. **Numerical** introduction: (← Student activity)
   a. Equation given $\frac{dy}{dx} = -\frac{x}{y}$.
   b. Each student needs to draw the slope field at the points.
   c. Meanwhile: Teacher sets two (or more) transparencies that he passes between students, and they need to draw on it one segment each.
   d. Then, we compare all slides together.

4. **What do slope fields show us?** (← Teacher activity)

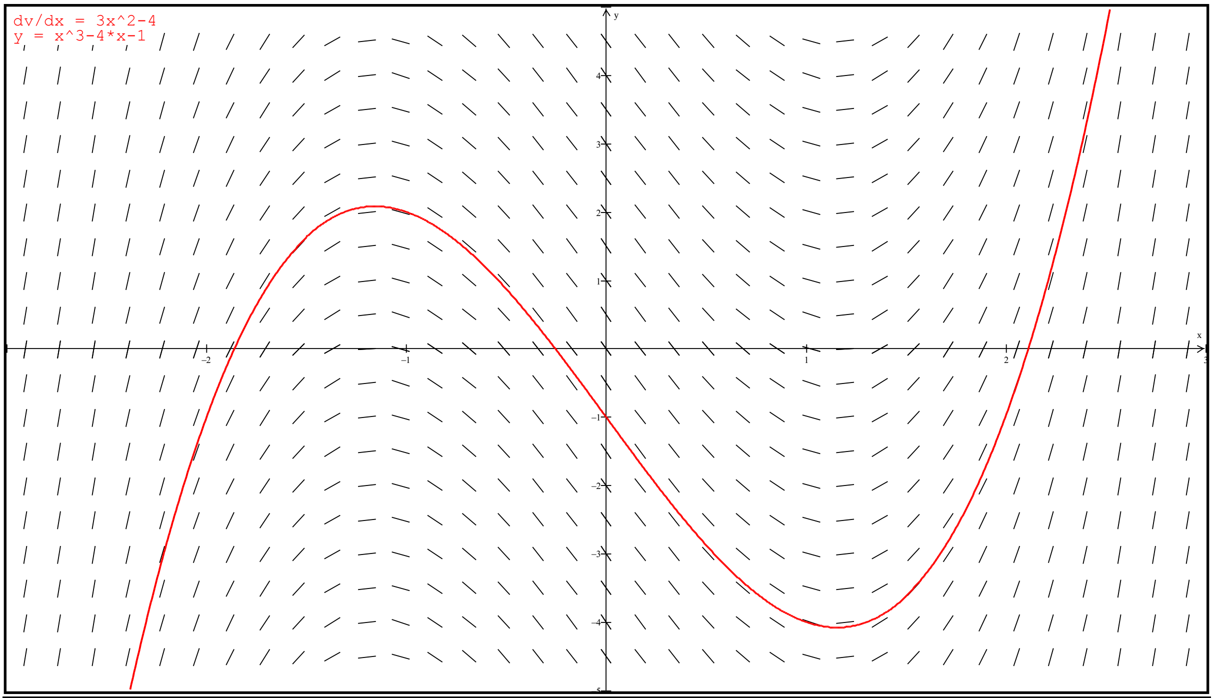
Consider the equation

$$\frac{dy}{dx} = 3x^2 - 4$$

with initial condition $y = -1$ at $x = 0$.

Draw the slope fields.

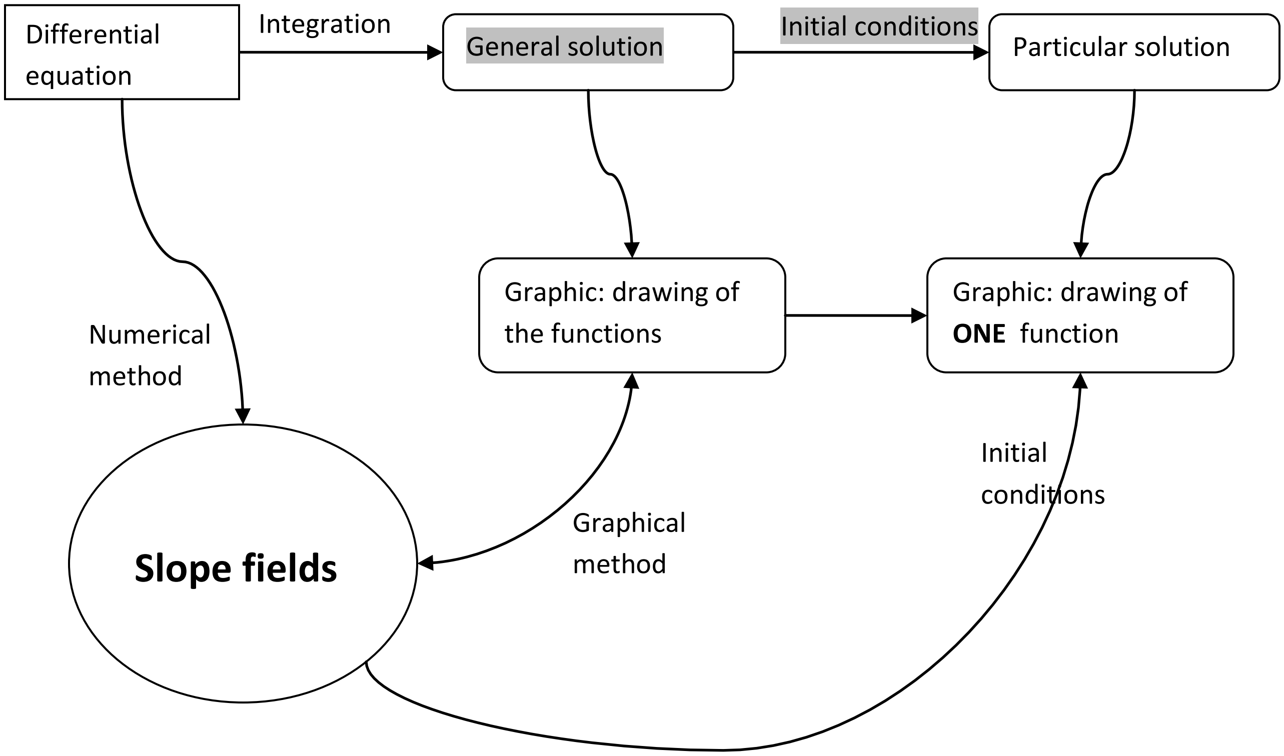
Then, start from the one given point, $y(x = 0) = -1$, and draw a curve.



<u>**Recap:** The solution curves are 'hiding' in the slope fields. Given one point on the solution curve, you can draw the whole solution!</u>

5. **Wrap-up :** Graphic organizer of the different things we learned today:

   Maybe leave out the shaded parts, and let them fill it in. Also, omit some of the arrows (from slope field to one-function).



6. **Post Wrap-up:** Often (!) on the AP there's at least one-Q on slope fields: We'll start have those as warm-up.
7. **Homework:** Start work on homework!

(There's also one more page with the graph by itself, in case we want to copy it to a slide and put on board)

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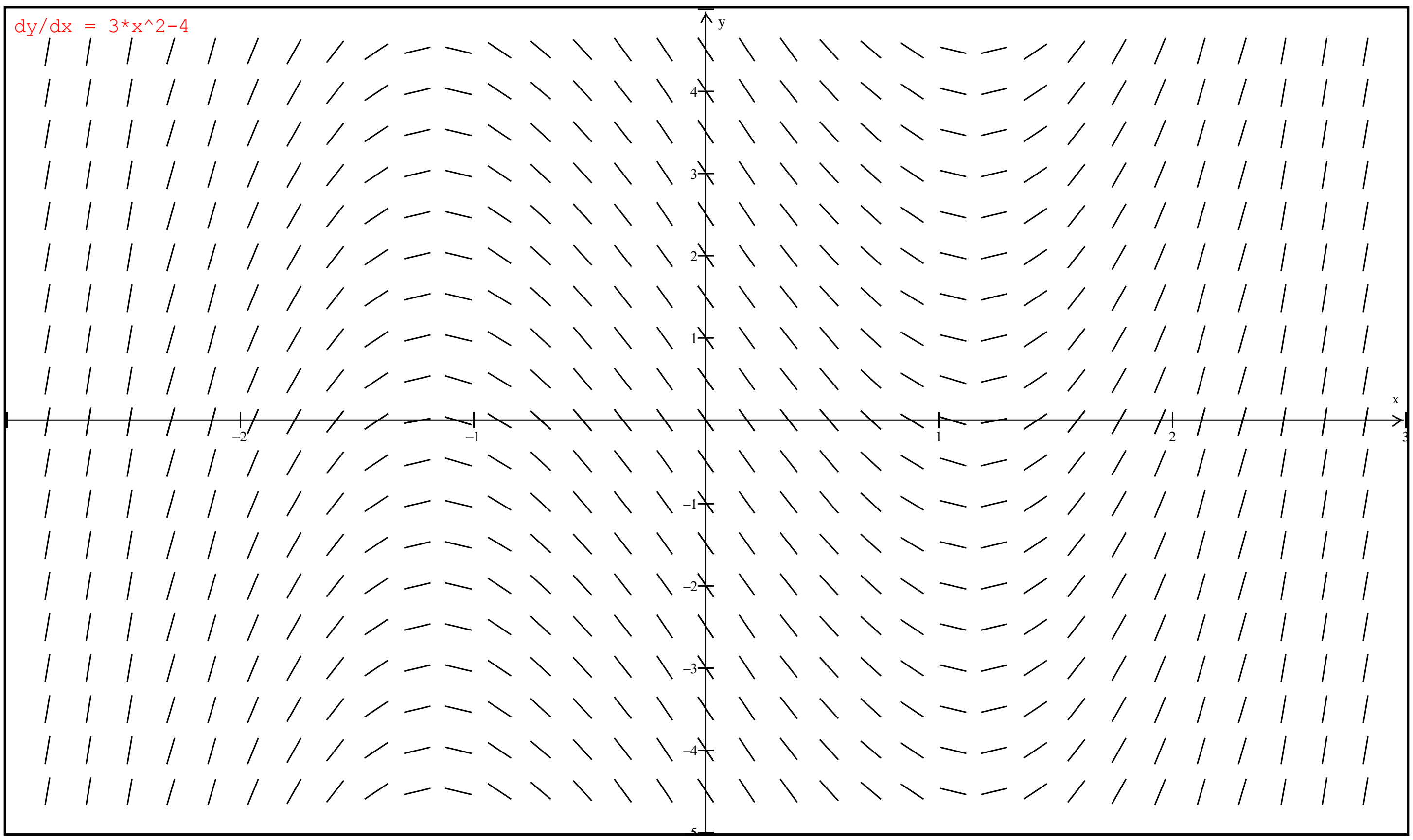
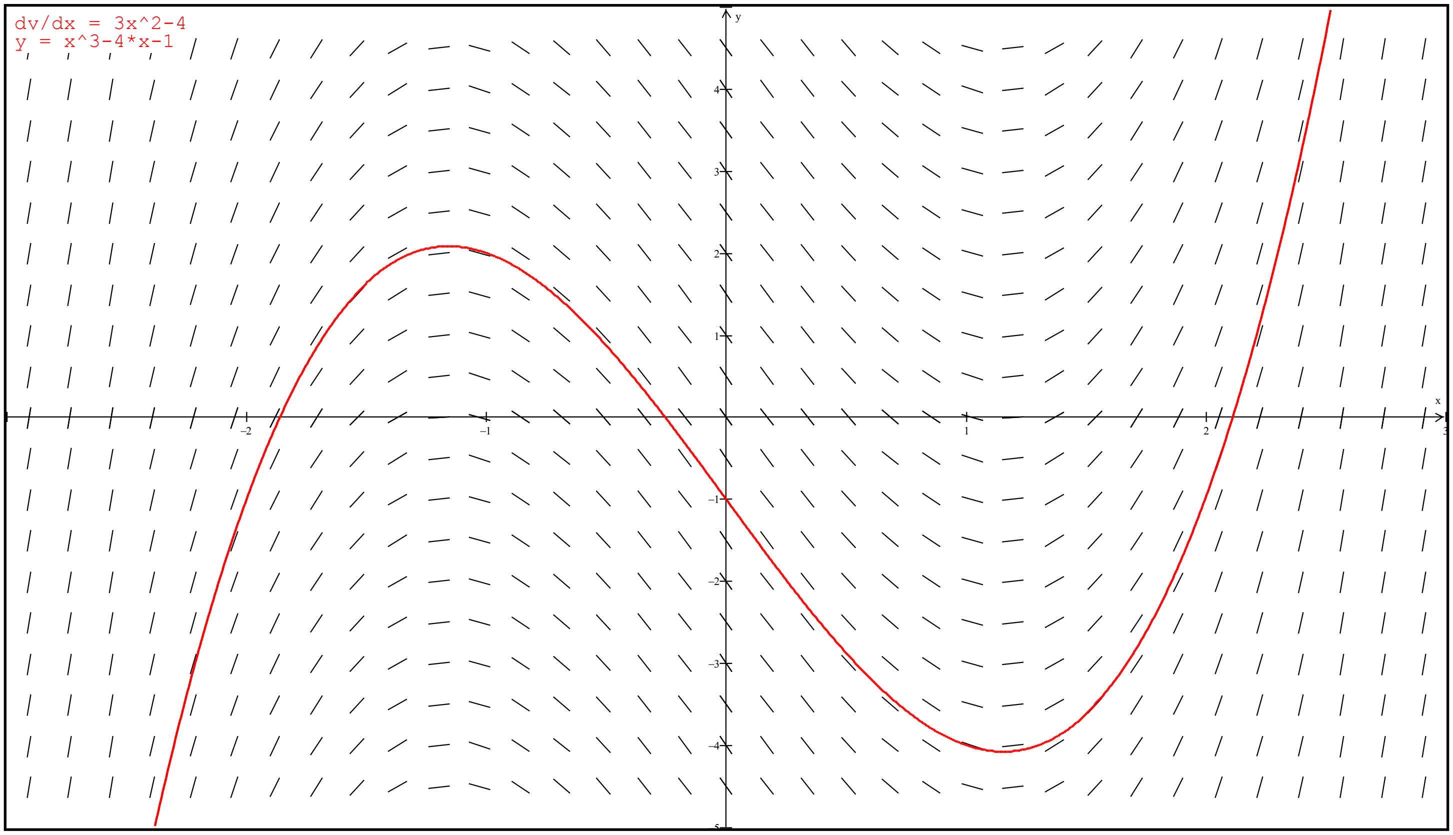

dy/dx = 3*x^2-4
y
x
4
3
2
1
−1
−2
−3
−4
−5
−2
−1
1
2
3



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