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Differential Equations: Calculus AB

Lesson Plan 6: Euler method (numerical solution).

Overview

This week has a lot of problem-solving, unit-project, and mostly deepening of the knowledge. They have all the tools they need for this unit. Now they need to know how to use those and gain more understanding of these tools.

Learning Objectives

- Euler method: Understanding and application.

Prior Knowledge needed

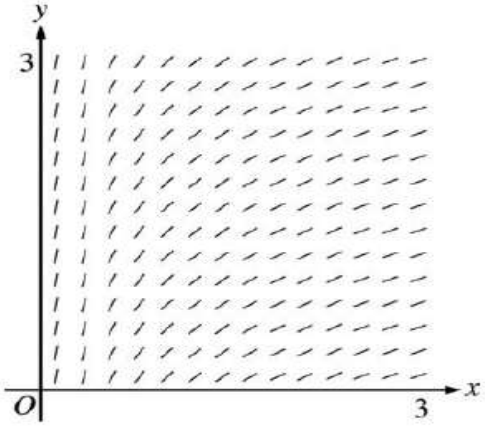
- The derivative expressed in the differential equation is the tangent to a solution curve.

Administration

1. **Unit-Project:** Bring every day your Unit-project page. Presentations next week.
2. **Homework:** Given for the whole week.

Instruction and activity

1. **Homework:** Tie loose ends with regard to homework checked over the weekend (Q85 p. 412 for example).
2. **Warm-up and review problem (from AP):**
   (2008 Ab exam)



7. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$

The main point to convey here: The differential equation describes slopes which are the tangents to the solution curves.

3. **Putnam problem (extra credit – Q95 p. 412):**
   - **a.** Introduce the Putnam competition (see below).
   - **b.** Go over the question, explain how to attack it, then dissect it, explain terms, and finally emphasize the notion that the differential equation describes the SLOPE of the tangent to the solution curve in a point. Do not completely solve: Give them the chance to do it.
   - **c.** Emphasize the need for 'problem solving' skills.

William Lowell **Putnam** Mathematical Competition
http://en.wikipedia.org/wiki/William_Lowell_Putnam_Mathematical_Competition

The William Lowell Putnam Mathematical Competition, often abbreviated to the **Putnam Competition**, is an annual mathematics competition for under graduate college students of the United States and Canada, awarding scholarships and cash prizes ranging from $250 to $2,500 for the top students and $5,000 to $25,000 for the top schools. The competition was founded in 1927 by Elizabeth Lowell Putnam in memory of her husband William Lowell Putnam, who, while alive was an advocate of intercollegiate intellectual competition. The exam has been offered annually since 1938 and is administered by the Mathematical Association of America.

The Putnam competition now takes place on the first Saturday in December, and consists of two three-hour sittings separated by a lunch break. Each competitor attempts to solve twelve problems, ***which can typically be solved with only basic knowledge of college mathematics but which require extensive creative thinking.***

The following table lists Teams finishing in Top Five (as of 2007 competition):

Top Five	Team (s)
53	Harvard
38	MIT
28	Caltech
26	Princeton
18	Toronto
17	Waterloo
12	Duke
11	Chicago, Washington U in StL, Yale
9	UC Berkeley, Cornell
6	Stanford

4. **Euler method: Solving with example**

We'll explain it using an example:

Differential equation: $y' = 2x$; Initial condition: $(0,1)$.

(The exact solution is $y = x^2 + 1$).
We'll do it numerically, starting from the given point.

- Initially do only the first 4 rows. Then, ask the students to do it for h=0.2.
- Draw it as you go, the approximate and exact solution. Note thePiece-wise linear segments in the approximation.

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
y'	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
y , h=0.1	1	1	1.02	1.04	1.07	1.11	1.16	1.22	1.29
y , Exact	1	1.01	1.04	1.09	1.16	1.25	1.36	1.49	1.64
y , h=0.2	1		1		1.04		1.12		1.18

- Better approximation as 'h' is smaller.

5. **Wrap up:**
   a. **Numerical method.**
   b. **The main points:** understand 'how' and 'why' it works.
6. **Start working on Homework:** P. 411: 75, 76, 78.

====End====