

Differential Equations: Calculus AB

Lesson Plan 7: Separation of Variables.

Overview

(As in 6): This week has a lot of problem-solving, unit-project, and mostly deepening of the knowledge. They have all the tools they need for this unit. Now they need to know how to use those and gain more understanding of these tools.

Learning Objectives

- Formally introduce the method of Separation of Variables.

Prior Knowledge needed

They already know this method. The purpose here is to strengthen their usage of it, and give it a formal name.

Administration

1. **Unit-Project:** We'll start working on it today.
2. Extra credit 'solve an AP question' on Thursday.
3. Next week presentation and review.

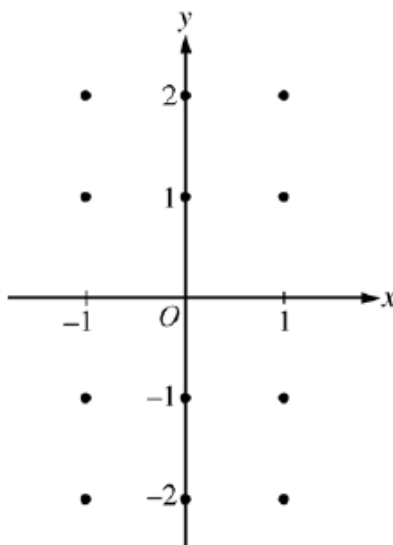
Instruction and activity

1. **Warm-up and review problem (from AP).**

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6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



- (b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

And / Or (Both are examples of separation of variables):

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6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

2. **Putnam problem (extra credit):** solve?!? (maybe at the end of the lesson, to allow continuity from the previous question into separation of variables.)

3. Separation of variables:

- a. They already know it and use it !!
- b. Maybe do one more involved question: Example 2 page 422 (the only drawback of that one is that it is not an explicit solution)

$$xydx + e^{-x^2}(y^2 - 1)dy = 0 \quad \text{with initial condition } y(0) = 1$$

4. Wrap-up :

- a. You've been using it all the time: Just be careful how you do it.
- b. Whenever possible: Check your results using different facets of the problem.

5. Working on unit project?!?

6. Homework: P.429: 6,7,11,20,21,23,25,26.

====End====