

Differential Equations: Calculus AB

Lesson Plan 9: Logistic Equation.

Overview

(As in 6,7 and 8): This week has a lot of problem-solving, unit-project, and mostly deepening of the knowledge. They have all the tools they need for this unit. Now they need to know how to use those and gain more understanding of these tools.

Learning Objectives

- Get familiar with the Logistic equation.

Prior Knowledge needed

The students should have learned in the previous lesson about exponential solutions.

Special Materials

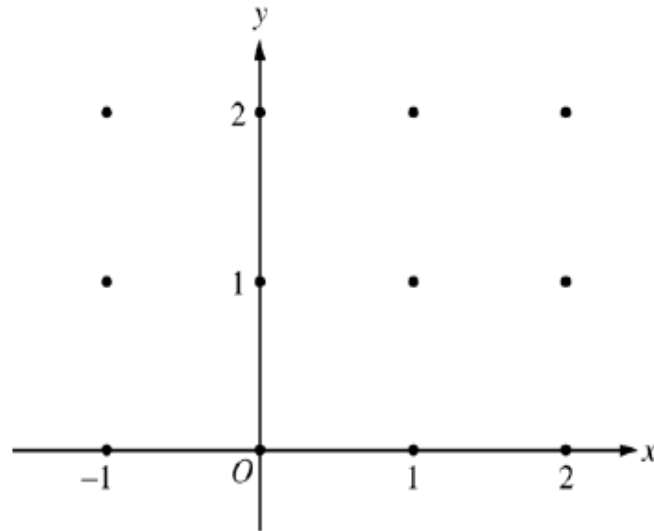
None.

Instruction and activity

1. **Warm-up and review problem (from AP).**
(This will be done as an extra-credit quiz)

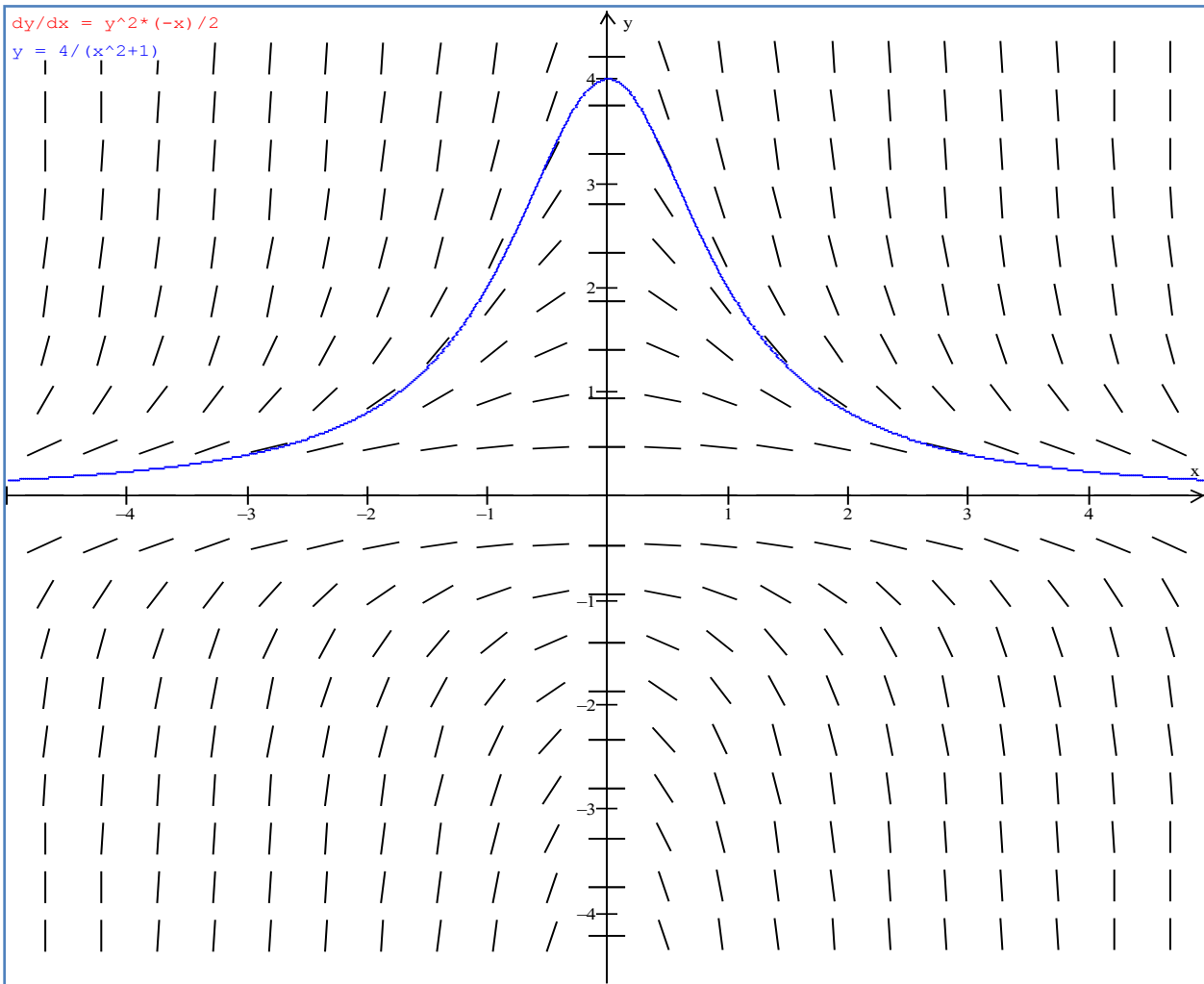
6. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (Note: Use the axes provided in the test booklet.)



- (b) Write an equation for the line tangent to the graph of f at $x = -1$.
- (c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

Solution: (graph + analytic) below.



2. Logistic equation:

Just to get them the name and how it looks like. Derivation is in the book.

The equation:

$$\frac{dy}{dx} = ky \left(1 - \frac{y}{L}\right)$$

K – growth factor.

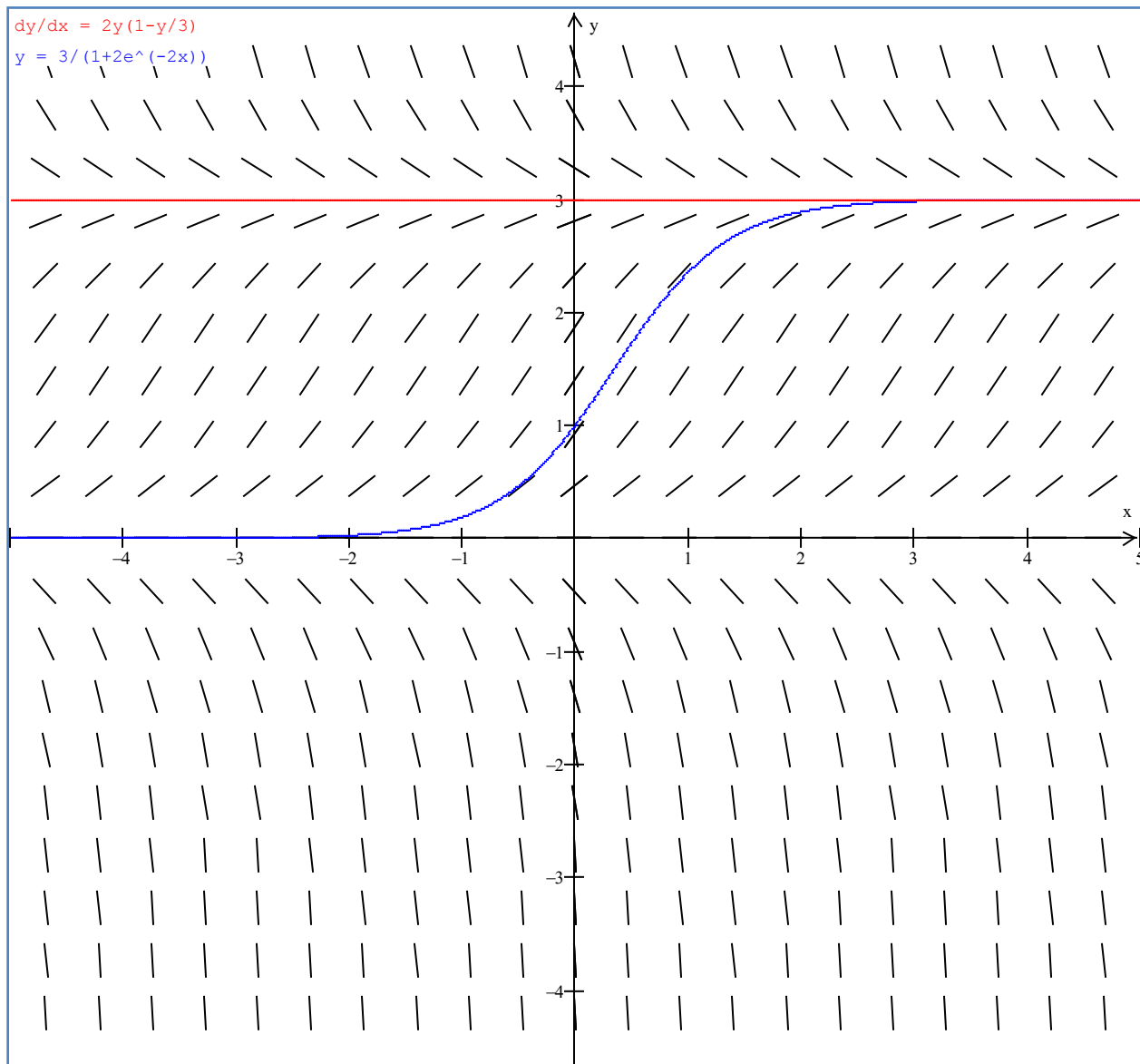
L – carrying capacity.

This suggests an exponential growth (ky element), but with carrying capacity (L).

And the solution:

$$y = \frac{L}{(1 + be^{-kt})}$$

See drawing below: b is according to initial value.



3. **Work on Unit project:** We'll check solutions using WinPlot.

4. **Wrap-up :** Two options.

- a. We mentioned a few concepts. Write down what the connection between those is:
 - i. Differential equation
 - ii. Solution curves
 - iii. Slope curve
 - iv. Particular solution

OR

- b. Which was the hardest subject for you in this unit so far (question and item) ? Why? Which was the easiest?

2. **Homework:**

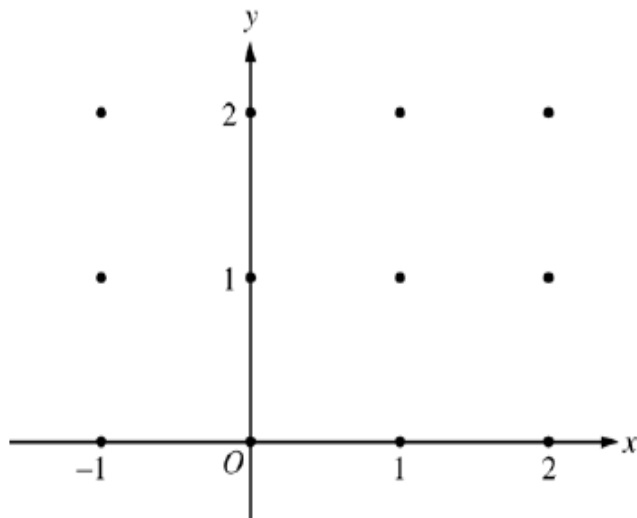
a. Write three question (+solutions) for the test: Easy, Medium, Hard.

====End (almost: Printout worksheet below)====

Name: _____

6. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (Note: Use the axes provided in the test booklet.)



(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

Solution:

$$dy/dx = y^2 \cdot (-x)/2$$

$$y = 4/(x^2+1)$$

