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Lesson Plan (Outline):
Hands-on geomtry proofs using $1-Origmai.

(Partly based on ideas from: **"Unfolding mathematics with Unit Origami"**, by Betsy Franco.)

Overview

This lesson can be used for a variety of levels. As indicated below, it can be extended to encompass various concepts, depending on the available time and the level of the course.

This lesson plan contains:

- Brief lesson-plan description.
- Analysis of the lesson plan using Van-Heili model.
- '$20 bill' printout page.
- PowerPoint slides, which are in a separate file. ← Main activity description.

Learning Objectives (very flexible: Depending on time and level).

- Familiarity with shapes: Rectangles, squares, triangles.
- Definition of Square, right-angle triangle, isosceles and equilateral triangle.
- Congruent triangles.
- 'Hands on' proofs.
- Angle chasing, parallel lines.
- Two column proofs.

Prior Knowledge needed

See learning objectives: It ranges from basic familiarity with shapes, to definitions, and into geometry and finally formal proofs.

Materials needed

Rectangle piece of paper: possible letter-size, or dollar size, and all variations in between. I used a letter-size page, printed to enlarged $20 bills on it, and cut it in the middle. Thus, students got to fold 'pretend' money, which is in a big-enough size to enable easy handling.

Available are power-point slides that explain this lesson plan, and can be used during class.

Instruction and activity

(See following in the Powerpoint slides)

1. **Hook** (possible) – An unfolded consumer-box. To discuss the marvels of geometry in real world applications.
2. **Warm up: Making a square out of rectangle** –Idea of triangles, what is a square, and so on…
    a. **Build:** Ask students to find it on their own. Compare their results with neighbors.
    b. **Discussion**: How do we know it is a square?
3. **Folding a dollar-bill into equilateral triangle** – The main activity.
    a. **Build:** Guide students through the steps, preferably using power-point slides.
    b. **Discuss:** Is this an equilateral? How do you know?
        i. **One way:** put your neighbor's triangle on top of it, and rotate, and see all angles/sides are the same.
        ii. **Other ways:** Fold yours to see the sides/angles are congruent.
    c. **Unfold, and look at the creases:**
        i. **Counting triangles:** How many equilateral triangles do you see?
        ii. **Congruent triangles:** Why do we say 'congruent' rather than 'equal'?
        iii. **Which are congruent? ;** iv. **Chasing angles**. ; v. **Proofs**.
        iv. Can we prove the first triangle we created is an equilateral? What are the ways to attack it?
        v. Can we build the creases with Geometer's sketchpad?
    d. **Possible extension**:
        i. **If we took another bill** - Would we get an equilateral? The same equilateral? How many?
        ii. **Predict** - Can we know in advance what we will get?

4. **Wrap-up**: Squares, triangles, and their properties. Cool trick with a dollar-bill (not only for $1).Geometry is everywhere, has aesthetic value, and many applications!

====End (There's still lesson analysis and one more page, with the picture of $20 bills they can fold)====

Level 0 - Visualization

Level 1 – Analysis (attributes)

Level 2 – Informal Deduction

Level 3 – Deduction

Level 4 – Rigor

Origami geometry lesson plan: van Heili model

The Origami-geometry lesson plan starts with folding a rectangle into equilateral, and then examining the result and the process. In this short write-up we will describe how this lesson plan fits the van Heili model. The lesson plan (very brief format) is attached at the end, and the van Heili model is explained within the following text.

The van Heili model is a cognitive theory that describes the development of geometric understating in children. It describes 5-stages of geometric reasoning:

0. **Visualization phase** – *Student can only recognize shapes, without the ability to justify the identification*. In the lesson plan, the students are continuously asked for what shapes they see and recognize. Thus, for example, after the 3rd fold, students are asked 'What does this shape looks like?'. It is an equilateral already at this stage, but there's no proof or reasoning for that right now.
1. **Analysis stage** – *Students formalize attributes of shapes*. Thus, after arriving at the last stage of folding, students are asked 'Why do you think it is an equilateral?'. At this stage they usually come up with the suggestions 'All sides are equal', or 'All angles are equal'.
2. **Informal deduction** – *Compare geometric shapes, and construct simple proofs*. This is done once the shape is unfolded, and students look at the underlying structure of the folds. We start by counting triangles, chasing some angles, and then hand-waving some proofs.
3. **Deduction** – *Basing proofs on postulates: Two column proofs*. This is the highest level students usually practice at high-school. Once the folding is open, and all we have are the creases, the students can draw those on their own, and follow a proof methodology to cement the hand-waving proof done in the previous stage.
4. **Rigor** – *Being able to work in an abstract setting, such as non-Euclidean geometry for example*. It is possible to view the last task, that of drawing the origami folds on a SketchPad, as such a stage. The student removes the physical aspect of Origami, and all the folds are abstracted and need to be constructed using ruler-and-compass constructions. It is indeed a difficult stage for students.

To conclude, the 5-stages of the van Heili model are covered in this lesson plan, which in turn enables the lesson plan to be either conveyed at different levels or as one mix. If it is carried as one mix, it is advisable to carry the first parts individually (or in 2-person groups), and move to 4-people groups as it goes to the higher levels.

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E5
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SPECIMEN
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SERIES 2004
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TWENTY DOLLARS


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THE UNITED STATES OF AMERICA
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IN GOD WE TRUST
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